

SIMULATION OF THE THERMAL CONDITIONS OF RADIOELECTRONIC DEVICES USING AVERAGING METHODS

A. N. Salamatin, V. A. Chugunov,
O. V. Yartsev, and O. Yu. Mamontova

UDC 536.25.001.24:621.3.049

A possible realization of the system approach to the simulation of thermal fields in cassette radioelectronic devices is discussed, taking account of the multiscaling of their hierarchical structure by the successive application of averaging methods.

Due to the increase in power and increasing miniaturization of modern radioelectronic devices, the problem of predicting and controlling their thermal conditions becomes more and more pressing. A strict hierarchy of structures of radioelectronic devices with respect to the levels of structural components [a microcircuit (an integrated circuit, a large-scale integrated circuit), a board or a standard replacement element, a unit, etc.] results in a certain schematization in calculating their thermal fields based on the system approach [1]: the description of the thermal conditions of structural units at each hierarchical level in terms of averaged "background" thermal fields of the constituent elements without additional structural specification of them. Such an approach also has obvious advantages at the design stage [2], under conditions of uncertainty of the configuration of separate units of radioelectronic devices.

It is natural to formulate and construct the corresponding series of adequate mathematical models in the framework of general methods of homogenization [3-5] and averaging of heterogeneous systems [6, 7].

We address this issue using the example of the averaged description of stationary thermal conditions of the standard replacement element (see Fig. 1) as a basic structural unit of modern cassette radioelectronic devices. The equations defining detailed distributions of thermal flows q' and temperature t' in such a heterogeneous structure are of the form

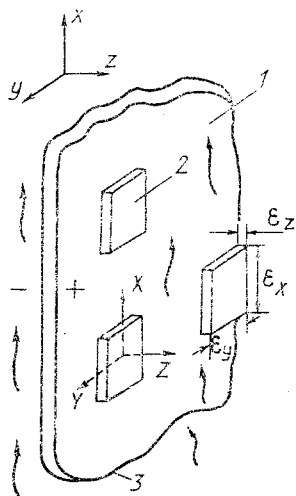


Fig. 1. Schematic representation of a fragment of the standard replacement element: 1) board; 2) microcircuits; 3) cooling medium (air).

$$\nabla \cdot \mathbf{q}'(p) = f'(p), \quad \mathbf{q}'(p) = -\lambda'(p) \nabla t'(p); \quad (1)$$

$$q_n'(p)|_{p \in S^\pm} = \alpha_\pm'(p)(t'(p) - t_0^\pm(M))|_{p \in S^\pm}.$$

A characteristic structural feature of the standard replacement elements (as well as of the other units of radioelectronic devices) is the large number of elements contained on it of a "lower" hierarchical level, microcircuits. This in its turn assures the smallness of the relative scale ε of the microcircuit dimensions and distances between them as compared to design dimensions of the board along the x-axis and y-axis, which we assume to have unit dimensions later on.

Let $\sigma'(p)$ be a characteristic function of the standard replacement element, and let $\omega_r(|M|)$ be an infinitely differentiable averaging kernel with averaging radius $r > 0$. We define, following [8], for an arbitrary field value $\varphi'(p; \varepsilon)$ on the standard replacement element the averaging operation:

$$\varphi(M) = \frac{1}{h} \int \omega_r(|M - \tilde{M}|) \sigma'(\tilde{p}) \varphi'(\tilde{p}) d\tilde{p}, \quad (2)$$

where $h = \int \omega_r(|M - \tilde{M}|) \sigma'(\tilde{p}) d\tilde{p}$ is the average thickness of the board with microcircuits ($h \sim \varepsilon$).

Then under conditions [3-5] by analogy with [9] we can affirm that for $\varepsilon \rightarrow 0$ the asymptotic representation of the averaged distribution $\varphi(M)$, if r is matched, and $\varepsilon/r \rightarrow 0$ is regular and represents a desired background (macroscale) description of the process under study without detailed fluctuations at distances of the order of ε .

The formal application of the averaging procedure (2) to Eqs. (1) yields

$$\nabla \cdot (h\mathbf{q}) = hf - q_s^+ - q_s^-. \quad (3)$$

As this takes place the average surface heat dissipation q_s^\pm from each side of the standard replacement element is defined by the equation

$$q_s^\pm = \int_{s^\pm} \omega_r(|M - \tilde{M}|) \alpha_\pm'(\tilde{p})(t'(\tilde{p}) - t_0^\pm(\tilde{M})) d\tilde{s}. \quad (4)$$

The problem of the construction of the closed mathematical model of the averaged thermal field of the standard replacement element on the basis of the balance Eq. (3) is therefore reduced to the derivation of additional equations, relating values q , q_s^\pm and t .

Solely from the considerations of simplicity of the exposition we assume later on that all the microcircuits, differing in power in the general case, have a similar construction, are positioned from one side of the board and are rectangular parallelepipeds with their sides parallel to the coordinate axes with dimensions $\varepsilon_x, \varepsilon_y, \varepsilon_z$ (see Fig. 1). To be specific we assume further $\varepsilon = (\varepsilon_x \varepsilon_y \varepsilon_z)^{1/3}$ and place in the center p^j with the coordinates $x^j, y^j, z^j, j = 1, 2, \dots$, of each microcircuit a special system of "quick" coordinates X, Y, Z : $p = p^j + \varepsilon P$, $P = (X, Y, Z)$. We introduce in agreement with [8, 10] for r and ε/r tending concordantly to zero a new type of averaged characteristics:

$$\begin{aligned} \underline{\sigma}(M; P) \underline{\varphi}(M; P) &= \rho^{-1} \sum_j \omega_r(|M - M^j|) \sigma'(p^j + \varepsilon P) \varphi'(p^j + \varepsilon P); \\ \underline{\sigma}(M; P) &= \rho^{-1} \sum_j \omega_r(|M - M^j|) \sigma'(p^j + \varepsilon P), \quad \rho = \sum_j \omega_r(|M - M^j|). \end{aligned} \quad (5)$$

These distributions describe in average (in the r -neighborhood of the point M) detailed processes around a particular, isolated microcircuit scaled to the scale of quick variables P .

From the point of view of additional closing relationships for the desired characteristics in (3) the values of quantities in (5) are first of all determined by the asymptotic relationship of the form

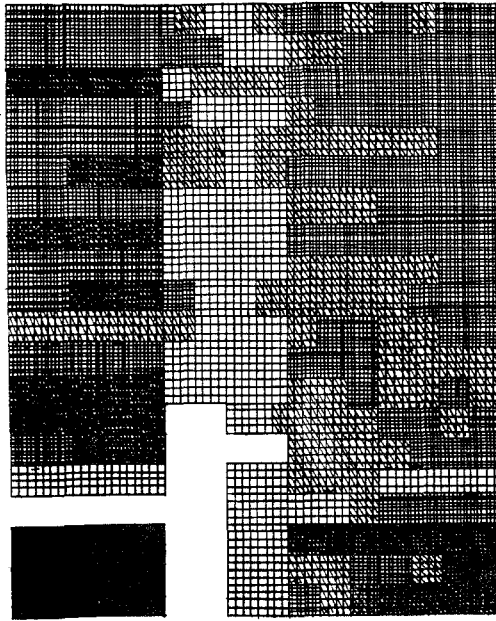


Fig. 2. The distributions of the conditional average density of microcircuits on a quarter of a standard replacement element. The degree of blackening corresponds to the variation in the value of the conditional average density from 0 (white tint) to 1 (black tint).

$$\varphi(M) \simeq \frac{\varepsilon}{h} \int \omega_{r/\varepsilon}(|N|) \underline{\sigma}(M; P) \underline{\varphi}(M; P) dP; \varepsilon \rightarrow 0, \quad (6)$$

between them and corresponding averages and determined by the direct check of [10].

We note that the spatial averaging used above in the construction of characteristics φ of type $\underline{\varphi}$ depending on the meaning and aims of the research conductor can be replaced, for example, at the design stage by the statistical averaging over the ensemble of possible versions of configurations of the standard replacement elements. Moreover, values (5) directly allow one to interpret [10] as conditional probability averages over a special ensemble of "copies" of the process under study, "taken" from the centers of microcircuits arranged in the neighborhood of the point M under consideration. When treated in this manner they coincide with the known analogies introduced in statistical mechanics [6]. Therefore, further considerations and results based on definitions (2) and (5) are much wider with respect to the application.

In the general case the problem of finding conditional average in the exact formulation is equivalent to the solution of the given system of Eqs. (1). However, the technological conditions for the assembly of microcircuits on a board inevitably set strict limits on their possible arrangement and thus predefine a certain quasi regularity in configuring the standard replacement elements. The consideration of the given feature of the construction of radioelectronic devices allows us to offer an approximate outline for constructing distributions $\underline{\varphi}(M; P)$.

Clearly, by definition $\underline{\sigma}(P) \equiv 1$ within the isolated microcircuit with center at the origin of coordinates ($P=0$) and in the board. The above-mentioned regularity in the structure of the standard replacement elements exhibits itself in the existence of a region around an isolated microcircuit in which there are no other radioelectronic elements and where $\underline{\sigma}(P)$ coincides with the characteristic function $\sigma'(p^j + \varepsilon P)$. At the same time, outside this region for $|Z| < \varepsilon_z / (2\varepsilon)$ the conditional average density of microcircuits on the standard replacement element can, as is seen from Fig. 2, vary nontrivially with the increase in $|X|$ and $|Y|$. Here it is natural to isolate the two extremen simulating situations: 1) the periodic distribution of microcircuits on the board, when the standard replacement element is obtained by duplicating along the two noncollinear directions in the XY-plane of a certain elementary cell of periodicity Ω containing one microcircuit, and $\underline{\sigma}(P) = \sigma'(p^j + \varepsilon P)$ for for any j and P; 2) the random arrangement of microcircuits, when for $|Z| < \varepsilon_z / (2\varepsilon)$ the value

$\underline{\sigma}(P)$ increase quickly from zero to its extreme average value $\rho\varepsilon_x\varepsilon_y$ with increasing $|X|$ and $|Y|$.

It is essential that in the second case it is also possible to introduce into consideration in the space of quick variables on the board the Ω neighborhood of the origin of coordinates containing one microcircuit and to approximate the conditional average density of the distribution of microcircuits for $|Z| < \varepsilon_z/(2\varepsilon)$ by the piecewise constant function:

$$\underline{\sigma}(P) \approx \begin{cases} \sigma'(p^j + \varepsilon P), & N \in \Omega; \\ \rho\varepsilon_x\varepsilon_y, & N \notin \Omega. \end{cases} \quad (7)$$

The average integral error of the given approximation is minimal [10] if the cell area mes $\Omega = \rho^{-1}\varepsilon^{-2}$, and its boundary $\underline{\sigma}(P)$ coincides with the curves of equal level $\underline{\sigma}(P)$.

The fluctuations of the conditional average characteristics (5) are due to geometrical heterogeneities - microcircuits on the board. Consequently, their fields should replicate the properties of the distribution $\underline{\sigma}(P)$. Thus when the structure of the standard replacement element is periodic

$$\underline{t}(M; P) - t(M + \varepsilon N) \text{ is a periodic function with respect to } X \text{ and } Y. \quad (8)$$

For the case of the random arrangement of microcircuits the conditional averages outside the Ω neighborhood approach macroscale characteristics. In particular, from the continuity of the thermal flows, it follows directly that

$$\underline{q}_n|_{N \in \partial\Omega} \approx \frac{h}{h_0} q_n|_{N \in \partial\Omega} \quad (h_0 = h - \rho\varepsilon^3). \quad (9)$$

Thus the problem of the construction of a closed mathematical model of the averaged description of the thermal conditions of the standard replacement element is reduced to determining the fields of the conditional average characteristics within the cell Ω , where σ , at least approximately, coincides with the characteristic function. The latter allows us to obtain directly

$$\begin{aligned} \bar{\nabla} \cdot \underline{q}(P) &= \varepsilon \underline{f}(P), \quad \underline{q}(P) = -\varepsilon^{-1} \underline{\lambda}(P) \bar{\nabla} \underline{t}(P); \\ \underline{q}_n|_{P \in S^\pm} &= \underline{\alpha}_\pm(P) (\underline{t}(P) - t_0^\pm)|_{P \in S^\pm}; \quad N \in \Omega \end{aligned} \quad (10)$$

from the general Eqs. (10) on the basis of definition (5).

In the derivation of (10) it was assumed in addition that $\underline{\lambda}(P) = \lambda'(p^j + \varepsilon P)$ and $\underline{\alpha}_\pm(P) = \alpha'_\pm(p^j + \varepsilon P)$ for any j .

It is clear that specific features of configuring the standard replacement elements in problem (10) are taken into account by the shape of the cell Ω and by the appropriate choice of the boundary conditions on $\partial\Omega$ of the form (8) or (9).

Correspondingly, Eq. (6) yields all the necessary defining relations:

$$\begin{aligned} \underline{t}(M) &= \frac{\rho\varepsilon^3}{h} \int dZ \int_{\Omega} \underline{\sigma}(M; P) \underline{t}(M; P) dX dY, \\ \underline{q}(M) &= \frac{\rho\varepsilon^3}{h} \int dZ \int_{\Omega} \underline{\sigma}(M; P) \underline{q}(M; P) dX dY, \\ \underline{q}_S^\pm(M) &= \rho\varepsilon^2 \int_{S^\pm(N \in \Omega)} \underline{\alpha}_\pm(P) (\underline{t}(M; P) - t_0^\pm(M)) dS. \end{aligned} \quad (11)$$

The factorized system of Eqs. (3), (8)-(11) contains a small parameter ε and admits different asymptotic representations depending on the conditions of heat exchange of the standard replacement element with the cooling medium, which are characterized by the Biot and Péclet numbers:

$$\text{Bi} = \alpha l^2 / (h\lambda), \quad \text{Pe} = c_p G l / (h\lambda).$$

For a representative class of modern radioelectronic devices a range of values of Bi and Pe of 10^2-10^3 is typical, which corresponds to the asymptotic process Bi, Pe $\rightarrow \infty$ for $\varepsilon \rightarrow 0$. For this case the macroscale effects of heat exchange on the standard replacement element are insignificant, and with a relative error of the order of $O(\text{Bi}^{-1} + \text{Pe}^{-1})$ the terms on the left-hand side of Eq. (3) can be neglected. Besides, for $\varepsilon \text{Bi} \gg 1$ irrespective of the arrangement of microcircuits on the board from (8) and (9) we have

$$q_n|_{N \in \partial \Omega} = 0. \quad (12)$$

The investigation of the corresponding problem (10)-(12) for the conditional averages allows us, by taking account of its linearity, to justify strictly the equations

$$q_S^\pm = \alpha_\pm \beta_\pm (t - \gamma t_0^\pm - (1 - \gamma) t_0^-) \mp \alpha_0 (t_0^\pm - t_0^-), \quad (13)$$

in which $\alpha_\pm = \rho \varepsilon^2 \int_{S^\pm(N \in \Omega)} \alpha_\pm(M) dS$, and the coefficients α_0 , β_\pm , and γ are expressed through the solutions of the particular problems and are functions of the structural and thermophysical parameters of the board and microcircuits. Indeed, if we designate by u the solution of problems (10)-(12) for $t^\pm = 0$, and we designate by \underline{v} the solution of the same system for $f=0, t_0^\pm=1, t_0^-=0$, then it can be shown that

$$\beta^\pm = u^\pm/u; \quad \gamma = v; \quad \alpha_0 = \alpha_- v^- = \alpha_+ (1 - v^+), \quad (14)$$

where

$$u^\pm = \frac{\rho \varepsilon^2}{\alpha_\pm} \int_{S^\pm} \alpha_\pm \underline{u} dS; \quad v^\pm = \frac{\rho \varepsilon^2}{\alpha_\pm} \int_{S^\pm} \alpha_\pm \underline{v} dS;$$

$$u = \frac{\rho \varepsilon^3}{h} \int_{\Omega} u dV; \quad v = \frac{\rho \varepsilon^3}{h} \int_{\Omega} v dV.$$

In particular, if $\varepsilon_z \ll \varepsilon_x, \varepsilon_y$ and $h_0 \ll \Delta_x, \Delta_y$, then it is not difficult to find out that

$$u^+ = K_\varepsilon (2R_1 - 1)(8a + 1)/(16R_1); \quad u^- = K_\varepsilon (2R_3^- - 1)c/(2R_3^-); \quad (15)$$

$$u = K_\varepsilon (a\varepsilon_z + b\delta + ch_0)/(K_\varepsilon \varepsilon_z + K_\varepsilon \delta + h_0); \quad (16)$$

$$v = K_\varepsilon [(a + 1/8)\varepsilon_z + b\delta + (c + d)h_0]/(K_\varepsilon \varepsilon_z + K_\varepsilon \delta + h_0); \quad (17)$$

$$v^- = (2R_3^- - 1)(K_\varepsilon c R_3 + (1 - K_\varepsilon) R_1 R_3^-)/(2R_1 R_3^- R_3). \quad (18)$$

Here $R_1 = 0,5 + \lambda_1/(\alpha_1 \varepsilon_z)$; $R_3^- = 0,5 + \lambda_3/(\alpha_3^- h_0)$; $R_3^+ = 0,5 + \lambda_3/(\alpha_3^+ h_0)$; $R_3 = R_3^- + R_3^+$; $R = 0,5 + R_1 + \lambda_1 h_0 (0,5 + R_3^-)/(\lambda_3 \varepsilon_z) + \delta \lambda_1/(\varepsilon_z \lambda_2)$; $a = (8R_1 \times (R - R_1) - R)/(8R)$; $c = R_1 R_3^- \lambda_1 h_0/(R \lambda_3 \varepsilon_z)$; $K_\varepsilon = \varepsilon_x \varepsilon_y/(\Delta_x \Delta_y)$; $b = c \cdot (\delta \lambda_3/((h_0 \lambda_2) + R_3^- + 0,5)/R_3^-)$; $d = (1 - K_\varepsilon) R_3^-/(K_\varepsilon R_3)$.

Now for the final values of the criterion Bi ~ 1 , when $\varepsilon \rightarrow 0$, the coefficients β^\pm in Eqs. (3) and (13) become close to unity, and Eqs. (10) assume the form

$$\overline{\nabla} \cdot (\underline{\lambda}(P) \overline{\nabla} \underline{t}(P)) = 0; \quad q_n|_{P \in S^\pm} = 0; \quad N \in \Omega \quad (19)$$

and in conjunction with the boundary conditions (8) or (9) on the basis of (11) define the macroscale thermal flow on the standard replacement element in Eq. (3)

$$\overline{q} = -\Lambda \nabla t. \quad (20)$$

The coefficient of effective thermal conductivity Λ in the general case is a tensor and depends substantially on the nature of the thermal interaction of the microcircuits with the board.

For the case of small thicknesses of microcircuits and the board, the problem (8), (9), (14) can be reduced to the plane problem, which is solved by the variational method. As a result, for the periodic arrangement of microcircuits the tensor Λ has only diagonal components $\Lambda_{xx}, \Lambda_{yy}$, determined from the equation

$$\Lambda_{xx} = \lambda_3 \{1 + \Lambda_0 + \Lambda_0^2 A \operatorname{th}(B) / [1 + \Lambda_1 \operatorname{th}(B) \cdot \operatorname{cth} \sqrt{3C\Lambda_2}]\}, \quad (21)$$

where $\Lambda_0 = (\lambda_0 - \lambda_3)K_3/\lambda_{30}$; $\Lambda_1 = \lambda_3/\sqrt{\lambda_{30}\lambda_{03}}$; $\Lambda_2 = \lambda_{30}/\lambda_{03}$; $\lambda_{30} = \varepsilon_x \lambda_3/\Delta_x + (1 - \varepsilon_x/\Delta_x)\lambda_0$; $\lambda_{03} = \varepsilon_x \lambda_0/\Delta_x + (1 - \varepsilon_x/\Delta_x)/\lambda_3$; $\lambda_0 = (\lambda_3 h_0 + \lambda_2 \varepsilon_z E + \lambda_1 \delta D)/(h_0 + \delta D + E \varepsilon_z)$; $A = (\Delta_x - \varepsilon_x)/(\Delta_x K_\varepsilon \sqrt{3C})$; $D = (\lambda_1 \varepsilon_z + \lambda_2 \delta)/(\lambda_1 \varepsilon_z + 2\lambda_2 \delta + 2\lambda_2 h_0/3)$; $B = (\Delta_y - \varepsilon_y)\sqrt{3C}/\varepsilon_y$; $E = (\lambda_1 \varepsilon_z/3)/(\lambda_1 \varepsilon_z + 2\lambda_2 \delta + 2\lambda_2 h_0/3)$; $C = \varepsilon_y^2/[\varepsilon_x(\Delta_x - \varepsilon_x)]$. The expression for Λ_{yy} is obtained from (21) by interchanging indices x and y.

We note that the discrepancies between the results of calculations according to (21) and numerical data [11] do not exceed 1%.

The obtained equations (14)-(18), (21) clearly verify the conclusion [12] that the effective characteristics of heat exchange depend significantly on the design features of microcircuits and the board and their thermophysical properties.

Concluding the discussion of the results of the application of averaging methods to the simulation of the thermal conditions of the standard replacement elements we note first of all that they are easily extended to the case of configuring the standard replacement elements by a number of different types of microcircuits.

The realization of similar considerations "from the bottom up" at each structural level of the radioelectronic device results in the construction of the hierarchical sequence of models. For example, by supplementing Eqs. (3), (13), and (15) by the equations of convective heat exchange in the gaps between the standard replacement elements we obtain the system describing the detailed temperature fields in the module - the analog of system (1) for the standard replacement element. Applying the corresponding averaging procedure gives a macro-scale model of the background thermal conditions for the next structural level. The calculation of thermal fields in the radioelectronic unit is realized successively "from the top down" with the detailing of its structure. We emphasize also that the approach considered allows us to take account of thermal interaction of separate structural units at each hierarchical level of the radioelectronic device in terms of average thermal fields. In this case the conditional average thermal fields allow us to estimate the corresponding temperature fluctuations in its elements.

NOTATION

cp, specific heat capacity of the cooling medium; f, volumetric power of heat evolution; G, mass flow rate of the cooling medium over unit length of the gap between the standard replacement elements; h, the average thickness of the standard replacement element taking account of microcircuits; h_0 , the board thickness; l, characteristic design size of the board; M, projection of the point on the xy-plane; N, projection of the point P on the XY-plane; Ω , a cell, the neighborhood of a microcircuit; V, the volume of the cell Ω ; Δ_x, Δ_y , cell dimensions when the shape is rectangular; p, a point in space with the coordinates x, y, z; \underline{q} , the vector of the heat flow density; qS, intensity of heat exchange on the surfaces of the standard replacement element [defined in (4)]; P, a point in the space of quick variable with the coordinates X, Y, Z; s(S), surface of the standard replacement element (in the space of quick coordinates); t and t_0 , temperature of the standard replacement element and of the cooling medium; u, v, auxiliary functions defining effective coefficients of thermal resistances in (14); α , coefficient of heat exchange; α_0, β, γ , coefficient of heat exchange and coefficients of thermal resistances of the standard replacement element in (13); ε , relative characteristic size of microcircuits; $\varepsilon_x, \varepsilon_y, \varepsilon_z$, dimensions of microcircuits (defined in Fig. 1); $\lambda(\Lambda)$, coefficient of effective heat exchange; ρ , density of the arrangement of microcircuits on the board [defined in (5)]; σ , characteristic function of the standard replacement element; φ , an arbitrary field value; $\nabla(\bar{\nabla})$, gradient operator (in the space of quick variables). Indices: -, +, lower and upper side of the standard replacement element (defined in Fig. 1); ', detailed microscale characteristics; underscore, conditional average characteristics; 1, 2, and 3, microcircuit, substrate, and board, respectively; x(y), on x-axis (on y-axis).

LITERATURE CITED

1. G. N. Dul'nev, Heat and Mass Exchange in Radioelectronic Devices [in Russian], Moscow (1984).
2. I. P. Norenkov and V. V. Manichev, Systems of Automated Design of Electronic and Computational Apparatus [in Russian], Moscow (1983).
3. V. V. Zhikov, S. M. Kozlov, O. A. Oleinik, and Kha T'en Ngoan, Usp. Mat. Nauk, 34, No. 15, 65-134 (1979).
4. A. Bensoussan, J.-L. Lions, and G. Rapanicolao, Asymptotic Analysis for Periodic Structures, North-Holland, (1978).
5. N. S. Bakhvalov and G. P. Panasenko, Process Averaging in Periodic Media [in Russian], Moscow (1984).
6. Yu. A. Buevich and I. N. Shchelchkova, Continuum Mechanics of Monodispersed Suspensions. Conservation Equations, Preprint, Institute of Problems of Mechanics [in Russian], Izd. Akad. Nauk. SSSR, No. 72, Moscow (1975).
7. R. I. Nigmatullin, Foundations of the Mechanics of Heterogeneous Media [in Russian], Moscow (1978).
8. A. N. Salamatin, V. A. Chugunov, and O. V. Yartsev, Applied Problems of Mathematical Physics [in Russian], Riga (1988), pp. 119-128.
9. A. N. Salamatin, Izv. Vyssh. Uchebn. Zaved., No. 40, 66-70 (1985).
10. A. N. Salamatin, Mathematical Models of Dispersion Flows [in Russian], Kazan (1987).
11. J. L. Lions, Computational Methods in Mathematical Physics, Geophysics, and Optimal Control [Russian translation], Novosibirsk (1978), pp. 5-19.
12. L. A. Kozdova, Heat and Mass Exchange VII. Papers of VII All-Union Conference on Heat and Mass Exchange, Vol. 7, Heat Conduction (1984), pp. 34-39.